i'Tec_S14
November 2004

Keywords: Distribution, Residence time, Duration, Processing, Data processing.

Measuring residence time distribution by pulsed injection
Data processing

This datasheet describes a data processing method based on the use of particulate tracers incorporated by pulsed injection (Tecaliman 2004 a, b and c).

1. Calculating the number of particles in each age category

The number of particles in each age category is equal to the number of particles per kilogram of feedstuff multiplied by the quantity of product processed during the period between two analysed samples.

Example:

Particle age (minute) Ti	Sample No.	Sample weight (g) Pi	Number of particles		
			per sample	per 1000 g of feedstuff	per age category
			ni	Ni = (ni/pi)*1000	(Ni)*(166.670)
T0	0	226	0	0	
T1	2	218	0	0	
T2	4	215	0	0	
T14	28	232	0	0	
T15	30	275	1	3.64	607
T16	32	279	1	3.58	597
T42	84	270	8	29.68	4.987
T43	86	246	5	20.33	3.389

Table 1

Process output rate = 10,000 kg/h, period between 2 analysed samples = 1 minute, quantity of product processed between two samples =10,000/60 i.e. 166.67 kg.

2. Graph of residence time distribution

Residence time distribution is shown by frequency density curves (or external ages), cumulated frequencies and, where appropriate, standardised curves (Tecaliman 2004a)

2.1. Frequency densities

Example: Frequency density or external ages in an extended duration conditioner (Figure 1).

2.2. Cumulated frequencies

Example: Cumulated frequencies curve in a soaking vessel process (Figure 2).

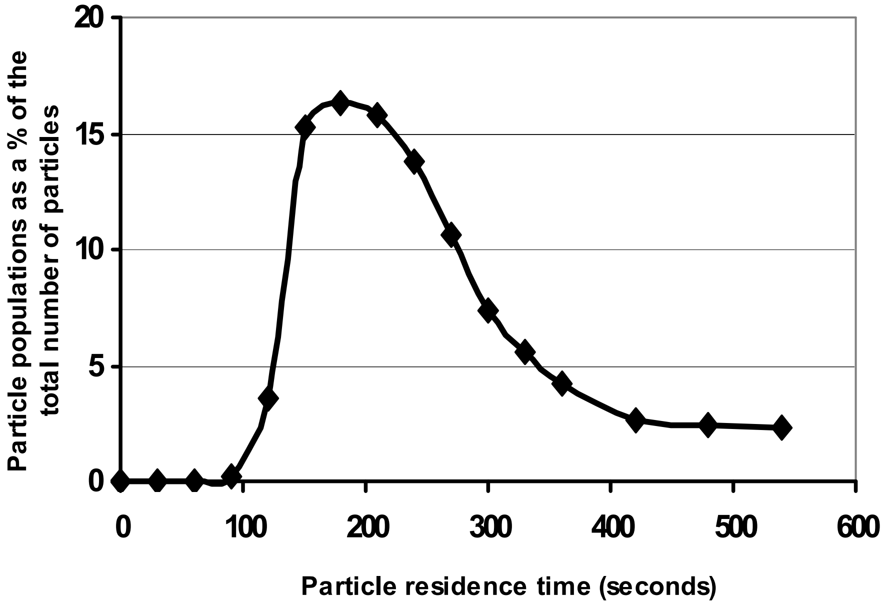

Figure 1: Function E (residence time frequency density)

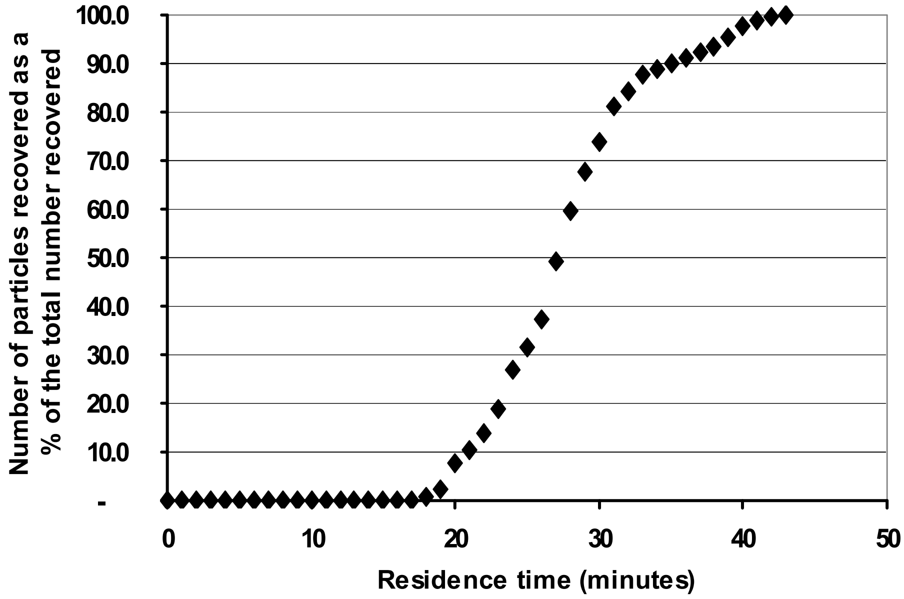

Figure 2: F function (Cumulated frequencies)

2.3. Standardised curves

Standardised curves can be used to compare the various RTD curves (Tecaliman 2004 a).
In this graph, the x and y axes are as follows:

- x-axis - reduced time,
- y-axis - reduced concentration.

Reduced time (Tri) is the ratio between the age of the particles at time Ti and throughput time Tp

Tri = Ti/Tp

where Tp theoretical throughput time (in seconds) is equal to the mass of product in the process (in kilograms) divided by the process output rate (kilogram/second).
Reduced concentration (Cri) is the concentration in tracer (Ni) at time Ti, divided by the equivalent concentration (Ceq), i.e. the concentration that the tracer would have if it was distributed evenly throughout the process:

Cri = Ni/Ceq
with
Ceq = N/Ma

where N is the total number of particles injected and Ma is the mass of product in the process at tracer injection time To.

Example:
The throughput Time measured during the tests was 181.8 seconds (Tp)
The Mass of product in the process was 676.7 kg (Ma)
The Number of particles injected was 1,512,000.
Equivalent Concentration at time Ti: Ceq = Ni/Ma = 223.4 p/100 g of feedstuff

Ti or particle age category (seconds)	Reduced time Tri = Ti/Tp	Number of particles in each age category and in 100 g of feedstuff	Reduced concentration Ni/Ceq
T0	0	0	0
T30	0.17	0	0
T60	0.33	1	0.0045
T90	0.50	322	1.4411
T120	0.66	423	1.8931

Table 2

3. Calculating the variables that typify RTD

Technical datasheet No.53 (Tecaliman 2004a) gives the meaning of these variables. Table 3 and **Table 6** give examples of how these variables are calculated.

3.1. General case

3.1.1. Tracer recovery rate

This equals the percentage of particles recovered from the actual number of particles injected.
The actual number of particles injected being equal to the actual mass of tracer injected multiplied by the number of particles in each gram of tracer.
Example:

Number of particles per gram of tracer	2,084
Quantity of tracer injected	412 grams
Number of particles injected	858,608
Number of particles recovered	817,040
Recovery rate %	95

Table 3

3.1.2. Mean particle residence time in the mixer-reactor

The mean residence time is equal to:

$$\overline{X} = \frac{\sum TiNi}{N}$$

where $N = \sum Ni$ is the total particle population

Example: 28 minutes in **Table 6**.

3.1.3. Median time

Using cumulated frequencies as a base, the median value is determined by selecting the 2 categories that bound the 50% population; it is assumed that all individuals are evenly distributed throughout this interval, and then the age of the 50% population is interpolated.
Example:
Table 4 and **Table 6** suggest that the median time should lie between the 26 and 27-minute age categories.

Age category or residence time (minute)	Cumulated frequency (%)
B = T26	D =46.66
A =T27	C = 56.67

Table 4

The median time is therefore equal to:

((A-B)x(50.00-D)/C-D))+D = ((27-26)x (50.00-46.66)/(56.67-46.66))+46.66 = 26.3 minutes

3.1.4. Dominant value or mode

This corresponds to the most frequent residence time.
Example: 27 minutes in **Table 6**.

3.1.5. The period between the time where 84 and 16% of particles were resident in the process (T68)

This is calculated based on the cumulated data, taking the times (T16% and T84%) needed to ensure that 16% and 84% of the total particle population has been through the process; time calculated by interpolation as for median time.

T68 = T 84% - T 16%

Example:
Table 6 shows that:

- time T16% lies between the 21 and 22-minute age categories, and is equal to 21.6 minutes.
- time T84% lies between the 32 and 33-minute age categories, and is equal to 32.5 minutes.

giving T68 = 32.5-21.6 = 10.9 minutes.

3.1.6. Minimum residence time

This is the time needed to record the first tracer particle exiting the process.
Example: 15 minutes in **Table 6**.

3.2. Normal distribution of residence times

3.2.1. Variance of the residence time distribution function

Variance S2 is equal to:

$$S^2 = \frac{\sum Ni\left(Ti - \overline{X}\right)^2}{N-1}$$

Example: 27.7 in **Table 6**.

3.2.2. Skewness coefficient for the residence time distribution curve, α3

$$\alpha 3 = \frac{m_3}{S^3}$$

where m_3 is the 3rd order moment and S is the standard deviation.

$$m_3 = \frac{\sum Ni\left(Ti - \overline{X}\right)^3}{N-1}$$

$$S = \sqrt{S^2}$$

3.2.3. Flatness coefficient for the residence time distribution curve, α3

$$\alpha 4 = \frac{m_4}{S^4}$$

where m_4 is the 4th order moment and S is the standard deviation.

$$m_4 = \frac{\sum Ni\left(Ti - \overline{X}\right)^4}{N-1}$$

4. Conclusion

Example:

Based on the data in Table 5 and **Table 6**, the following conclusion can be drawn:

The tracer recovery rate over a 43-minute sampling time is very high, i.e. 95%. The technique used to measure residence time distributions in this test campaign can therefore be considered reliable: choice of tracer, sampling technique and frequency. Note that 68% of the injected tracer particles were recovered between the times 21.6 and 32.5 minutes, i.e. over a 10.9-minute period.

The values for mean time, median time and mode were very similar, being 28, 26 and 27.8 minutes respectively. This suggests that residence time distributions are unimodal (there is a single particle throughput peak) and only weakly asymmetric. This was confirmed by the skewness coefficient value (0.6) which represents a creep in particle residence time distribution.

The minimum product residence time, from the soaker mixer input (MMA) to the press mixer input (MAL), was 15 minutes.

The value of the flattening coefficient (3.2) reveals that residence time distribution shows a sharpness similar to that of a normal distribution, i.e. the data is densely concentrated around the mode.

Tracer recovery rate	95%
Mean time	28 minutes
Median time	26 minutes
Mode (value with the highest frequency)	27.8 minutes
T 68%	10.9 minutes
Minimum processing time	15 minutes
Skewness coefficient (α3)	0,6
Flattening coefficient (α4)	3,2

Table 5

5. Bibliography

Tecaliman 2004a. Measuring treatment times - Concept of residence time distribution – i'Tec_S11 – november 2004.

Tecaliman 2004b. Measuring residence time distribution - Tracers– i'Tec_S12 – november 2004.

Tecaliman 2004c. Measuring residence time distribution - Protocol– i'Tec_S13 – November 2004.

Ti Age category (min)	Ni Population in each category (Number of particles)	Ni cum	Ti x Ni	Ti - mean	Ni*(Ti - mean)^2	Ni*(Ti - mean)^3	Ni*(Ti - mean)^4
T0	0.00	0.00	0.00	-28.00	0.00	0.00	0.00
T1	0.00	0.00	0.00	-27.00	0.00	0.00	0.00
T2	0.00	0.00	0.00	-26.00	0.00	0.00	0.00
T3	0.00	0.00	0.00	-25.00	0.00	0.00	0.00
T4	0.00	0.00	0.00	-24.00	0.00	0.00	0.00
T5	0.00	0.00	0.00	-23.00	0.00	0.00	0.00
T9	0.00	0.00	0.00	-19.00	0.00	0.00	0.00
T10	0.00	0.00	0.00	-18.00	0.00	0.00	0.00
T11	0.00	0.00	0.00	-17.00	0.00	0.00	0.00
T12	0.00	0.00	0.00	-16.00	0.00	0.00	0.00
T13	0.00	0.00	0.00	-15.00	0.00	0.00	0.00
T14	0.00	0.07	0.00	-14.00	0.00	0.00	0.00
T15	0.07	0.07	1.06	-13.00	11.94	-155.22	2,017.45
T16	0.00	0.07	0.00	-12.00	0.00	0.00	0.00
T17	0.00	0.56	0.00	-11.00	0.00	0.00	0.00
T18	0.49	1.96	8.86	-10.00	49.21	-491.94	4,918.13
T19	1.40	7.22	26.58	-9.00	113.27	-1,019.11	9,169.39
T20	5.26	9.92	105.17	-8.00	336.32	-2,689.72	21,510.95
T21	2.70	13.28	56.65	-7.00	132.08	-924.24	6,467.35
T22	3.36	17.96	73.92	-6.00	120.86	-724.87	4,347.35
T23	4.68	25.54	107.75	-5.00	117.00	-584.72	2,922.13
T24	7.58	29.98	181.85	-4.00	121.08	-484.01	1,934.79
T25	4.44	35.31	111.08	-3.00	39.92	-119.66	358.69
T26	5.33	46.66	138.52	-2.00	21.26	-42.46	84.81
T27	11.35	56.67	306.52	-1.00	11.29	-11.27	11.24
T28	10.01	64.40	280.14	0.00	0.00	0.00	0.00
T29	7.74	70.16	224.32	1.00	7.77	7.79	7.81
T30	5.75	77.13	172.59	2.00	23.07	46.20	92.52
T31	6.97	80.15	216.06	3.00	62.83	188.66	566.47
T32	3.02	83.52	96.66	4.00	48.39	193.69	775.24
T33	3.37	84.47	111.30	5.00	84.40	422.23	2,112.21
T43	0.39	95.48	16.97	15.00	88.84	1,332.79	19,995.17
Total	Number = N = 95		2,662.25		2,602.21	8,467.84	233,083.86

Mean	28.00	4th order moment	2,479.6
Variance	2.7	Flattening coeff.	3.2
3rd order moment	90.1	Skewness coeff.	0.6

Table 6